Dynamics Add-on: Momentum, FBDs and Graphs

H2 Physics crash course for force questions, collisions and graph interpretation

Big picture: dynamics has three equivalent languages.

$$\boxed{\sum F = ma} \qquad \boxed{F = \frac{\Delta p}{\Delta t}} \qquad \boxed{F\Delta t = \Delta p}$$

Force can be studied through acceleration, through momentum change, or through the area under a force-time graph.

Momentum: The "Amount of Motion"

Definition

$$\boxed{p = mv}$$

Momentum is mass times velocity. It is a vector, so direction matters.

A moving object is hard to stop if it has large momentum. That can happen because it has large mass, large velocity, or both.

Examples:

- A slow lorry can have large momentum because its mass is huge.
- A small bullet can have large momentum because its speed is huge.
- A ball moving right and a ball moving left can have momentum with opposite signs.

Sign convention

Always choose a positive direction first. For example, take right as positive.

$$\text{right-moving object: } p = +mv, \qquad \text{left-moving object: } p = -mv$$

This matters most when objects bounce, because velocity changes sign.

Common trap: if a ball bounces from rightwards to leftwards, its momentum change is large, not small. The velocity changes sign, so you must subtract with signs.

Change in momentum

$$\Delta p = p_{\text{final}} - p_{\text{initial}} = m(v - u)$$

If an object reverses direction, the signs do a lot of the work.

Mini-example: a 0.20 kg ball moving right at $10\,\mathrm{m\,s^{-1}}$ hits a wall and rebounds left at $8\,\mathrm{m\,s^{-1}}$. Take right as positive.

$$p_i = 0.20(10) = +2.0\,\mathrm{kg\,m\,s^{-1}}$$

$$p_f = 0.20(-8) = -1.6\,\mathrm{kg\,m\,s^{-1}}$$

$$\Delta p = -1.6 - 2.0 = -3.6\,\mathrm{kg\,m\,s^{-1}}$$

The negative sign means the impulse on the ball is leftwards. Magnitude: $3.6\,\mathrm{N\,s}$.

Impulse: Force Applied Over Time

Core idea

Impulse is the effect of a force acting for a time. It changes momentum.

$$\boxed{J = F\Delta t = \Delta p}$$

For changing force, use average force:

$$\boxed{J = F_{\text{avg}}\Delta t}$$

This explains airbags, bending knees when landing, and moving your hand backwards when catching a ball.

The momentum change needed to stop the object is fixed. If you increase the stopping time, the average force becomes smaller.

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t}$$

Newton's second law in momentum form

The deeper form of Newton's second law is:

$$\boxed{F = \frac{\Delta p}{\Delta t}}$$

If mass is constant:

$$F = \frac{\Delta(mv)}{\Delta t} = m\frac{\Delta v}{\Delta t} = ma$$

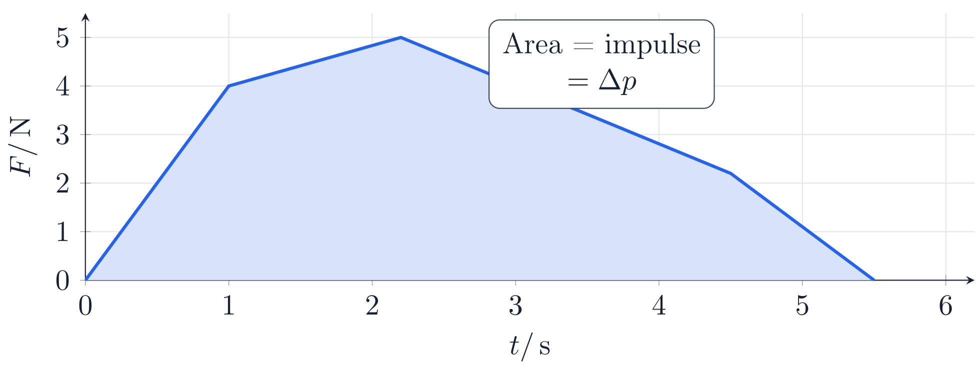
So $F = ma$ is a special case of force being the rate of change of momentum.

Force-Time Graphs

The key rule

$$\boxed{\text{area under force-time graph} = \text{impulse} = \Delta p}$$

The gradient of a force-time graph is usually not important at A-Level. The area is the main thing.



If the graph is rectangular:

$$J = F\Delta t$$

If the graph is triangular:

$$J = \frac{1}{2}(\text{base})(\text{height})$$

If the graph is irregular, estimate area by counting squares or use given average force.

Average force from graph

$$F_{\text{avg}} = \frac{\text{area under graph}}{\text{total time}}$$

This is the constant force that would produce the same impulse over the same time.

Impact intuition: a sharp tall spike means a large force over a short time. A wider lower graph can have the same area, meaning the same momentum change but less peak force.

Momentum Conservation

When is momentum conserved?

$$\boxed{\text{total momentum before} = \text{total momentum after}}$$

This is true for a system if the resultant external force on the system is zero or negligible during the interaction.

In collisions, the internal forces between objects are equal and opposite. They change each object's momentum, but the total system momentum stays constant.

Collision types

Type	Momentum	Kinetic energy
Elastic collision	Conserved	Conserved
Inelastic collision	Conserved	Not conserved
Perfectly inelastic collision	Conserved	Not conserved; objects stick together
Explosion	Conserved	Kinetic energy usually increases because stored energy becomes kinetic energy

Exam trap: momentum is conserved in collisions even when kinetic energy is not. Do not write "energy is lost" without saying it becomes internal energy, heat, sound, deformation, etc.

The standard collision equation

For two objects moving in one dimension:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

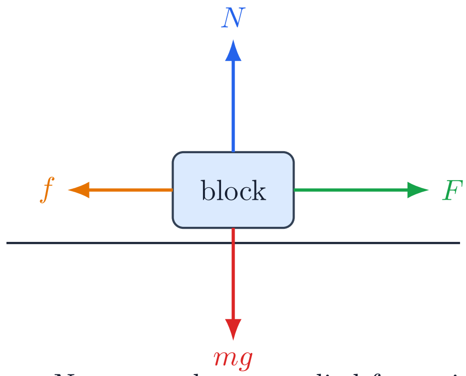
Use signs. If object 2 moves left, its velocity is negative.

Free-Body Diagrams: The Skill That Saves Marks

What an FBD is

A free-body diagram shows all forces acting on one chosen object. It does not show forces that object exerts on other things.

Rule: one diagram, one object. Every arrow must be a force acting on that object.



Horizontal surface: N up, mg down, applied force right, friction left.

The force checklist

For any object, ask:

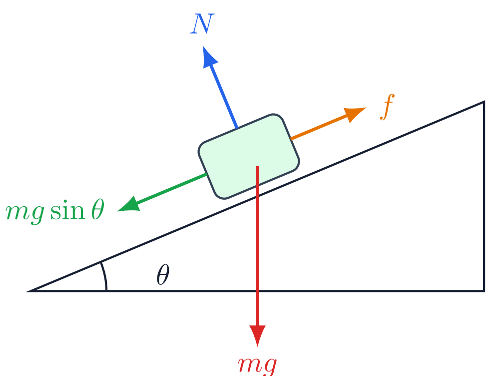
1. Is there mass near Earth? Draw weight mg vertically downward.
2. Is it touching a surface? Draw normal force perpendicular to the surface.
3. Is the surface rough? Draw friction along the surface, opposing sliding or tendency to slide.
4. Is there a string? Draw tension along the string, pulling away from the object.
5. Is there a push, pull, drag or thrust? Draw it in the stated direction.

What not to draw

Do not draw velocity, acceleration, displacement or momentum as forces. They are useful information, but they are not force arrows on an FBD.

FBDs on Slopes

The most important slope trick: resolve weight into components parallel and perpendicular to the slope.



For a block sliding down: $mg\sin\theta$ acts down the slope, friction f acts up the slope. Into the slope: $mg\cos\theta$, so usually $N = mg\cos\theta$.

Important: $mg\sin\theta$ and $mg\cos\theta$ are components of weight, not extra forces. In working, either draw mg and resolve it, or replace mg by its components. Do not count both the full mg and its components in the same force equation.

For a block sliding down a rough slope:

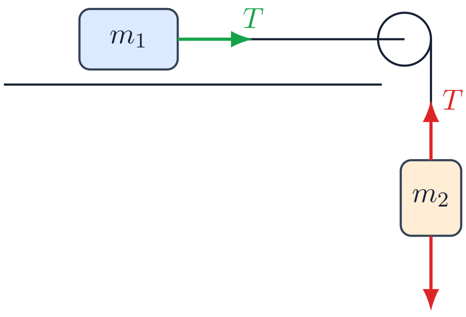
$$mg \sin \theta - f = ma$$

If $f = \mu N$ and $N = mg \cos \theta$:

$$mg \sin \theta - \mu mg \cos \theta = ma$$

FBDs for Connected Bodies

Connected-body questions usually involve two separate FBDs.



For m_1: usually $T - f = m_1 a$. For hanging m_2: usually $m_2 g - T = m_2 a$. Same string gives same tension; inextensible string gives same acceleration magnitude.

System shortcut

If you treat both masses as one system, tension is internal and cancels. This is useful for finding acceleration first. Then use one object's equation to find tension.

Graph Skills for Dynamics

The master table

Graph	Gradient means	Area means
Force-time	Usually not important	Impulse, $J = \Delta p$
Momentum-time	Force, $F = \Delta p / \Delta t$	Usually not important
Velocity-time	Acceleration	Displacement
Acceleration-time	Rate of change of acceleration; rarely used	Change in velocity
Force-displacement	Usually not important	Work done, $W = Fs$ if constant

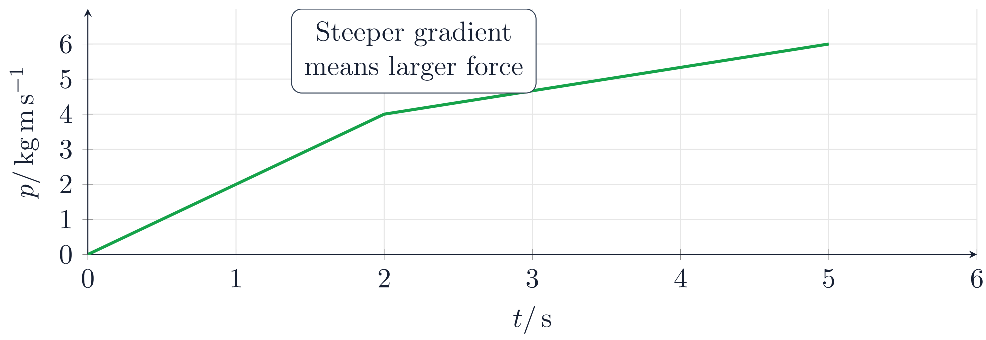
Momentum-time graph

Because force is rate of change of momentum:

$$F = \frac{\Delta p}{\Delta t}$$

So on a momentum-time graph:

$$\boxed{\text{gradient} = \text{force}}$$



Force-displacement graph

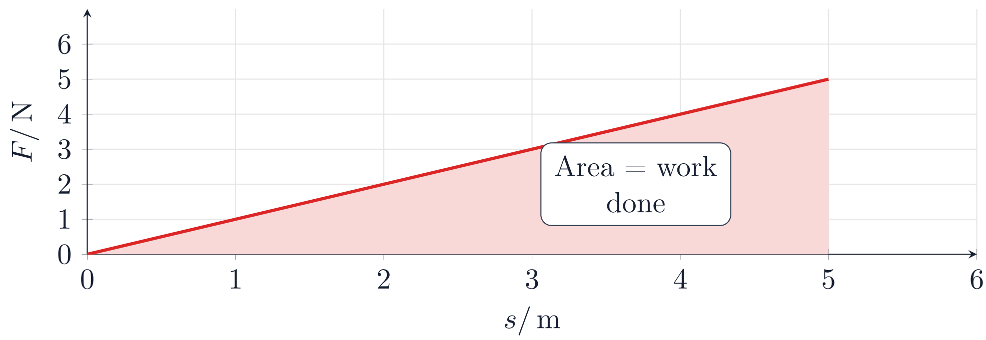
Force over distance is work:

$$W = Fs$$

For a graph:

$$\boxed{\text{area under force-displacement graph} = \text{work done}}$$

This links dynamics to energy.



How to Attack Exam Questions

For force/FBD questions

1. Choose one object.
2. Draw all forces acting on that object.
3. Choose a positive direction, usually direction of acceleration.
4. Resolve forces parallel and perpendicular to motion/surface.
5. Write $\sum F = ma$ in the direction of acceleration.
6. If another object is connected, write another equation and solve simultaneously.

For momentum/impulse questions

1. Choose a positive direction.
2. Write initial and final momenta with signs.
3. Find $\Delta p = p_f - p_i$.
4. Use $J = \Delta p$ or $F_{\text{avg}}\Delta t = \Delta p$.
5. For collisions, use total momentum before equals total momentum after.

For graph questions

First identify the axes. Then ask:

- Is it force-time? Area is impulse.
- Is it momentum-time? Gradient is force.
- Is it velocity-time? Gradient is acceleration; area is displacement.
- Is it force-displacement? Area is work.

Common Mistakes to Eliminate

- Do not put velocity or acceleration arrows on an FBD and call them forces.
- Do not draw Newton's third law pairs on the same object's FBD.
- Do not assume $N = mg$ on slopes or with vertical acceleration.
- Do not forget signs in momentum. Direction matters.
- Do not say momentum is conserved if there is a significant external force on the chosen system.
- Do not use graph gradient when the question needs area, especially for force-time graphs.

One-Page Mental Anchor

$\boxed{\sum F = ma}$ force over mass gives acceleration

$\boxed{F\Delta t = \Delta p}$ force over time gives impulse/momentum change

$\boxed{Fs = W}$ force over distance gives work/energy transfer

If you know what the force is acting over, you know the topic:

- force over mass $\rightarrow$ acceleration/dynamics
- force over time $\rightarrow$ impulse/momentum
- force over distance $\rightarrow$ work/energy